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XXII. ΚΟΣΚΙΝΟΝ ΕΡΑΤΟΣΘΕΝΟΥΣ.

O R.

The Sieve of Eratosthenes.

Being an account of his method of finding all the Prime Numbers, by the Rev. Samuel Horsley, F. R. S.

Read May 7,
1772.

A Prime number is such a one, as hath no integral divisor but unity.

A number, which hath any other integral divisor, is Composite.

Two or more numbers, which have no common integral divisor, besides unity, are said to be Prime with respect to one another.

Two or more numbers, which have any common integral divisor besides unity, are said to be Composite with respect to one another.

The distinction of numbers into Prime and Composite, is so generally understood, that I suppose it is needless to enlarge upon it.

To determine, whether several numbers proposed be Prime or Composite *with respect to one another*, is an easy Problem. The solution of it is given by Euclid, in the three first propositions of the 7th

book of the Elements, and is to be found in many common treatises of Arithmetic and Algebra. But to determine, concerning any number proposed, whether it be *absolutely* Prime or Composite, is a Problem of much greater difficulty. It seems indeed incapable of a direct solution, by any general method; because the successive formation of the prime numbers doth not seem reducible to any general law. And for the same reason, no direct method hath hitherto been hit upon, for constructing a Table of all the prime numbers to any given limit. Eratosthenes, whose skill in every branch of the philosophy and literature of his times, rendered his name so famous among the Sages of the Alexandrian School, was the inventor of an indirect method, by which such a table might be constructed, and carried to a great length, in a short time, and with little labour. This extraordinary and useful invention is at present, I believe, little, if at all, known; being described only by two writers, who are seldom read, and by them but obscurely; by Nicomachus Gerasinus, a shallow writer of the 3d or 4th century, who seems to have been led into mathematical speculations, not so much by any genius for them, as by a fondness for the mysteries of the Pythagorean and Platonic philosophy; and by Boethius, whose treatise upon numbers is but an abridgment of the wretched performance of Nichomachus*. I flatter myself therefore, that a succinct account of it will not be unacceptable to this learned Society.

* There are more pieces than one of this Nichomachus extant. That which I refer to is intitled *Εισαγωγή Αριθμητική*.

But before I enter expressly upon the subject, I must take the liberty to animadvert upon a certain Table, which, among other pieces ascribed to Eratosthenes, is printed at the end of the beautiful edition of Aratus published at Oxford in the year 1672, and is adorned with the title of Κοσμικὸν Ἐρατοσθένους. It contains all the odd numbers from 3 to 113 inclusive, distributed in little cells, all the divisors of every Composite number being placed over it, in its proper cell, and the Prime numbers are distinguished, so far as the table goes, by having no divisors placed over them. It hath probably been copied either from a Greek comment upon the Arithmetic of Nicomachus, preserved among the manuscripts of Mr. Selden in the Bodleian Library, in which, though the manuscript is now so much decayed as to be in most places illegible, I find plain vestiges of such a table *, which might be more perfect 100 years ago, when the Oxford Aratus was published; or else, from another comment, translated from a Greek manuscript into Latin, and published in that language, by Camerarius, in which a table of the very same form occurs, extending from the number 3 to 109 inclusive. It may sufficiently screen the editor of Aratus from censure, that he had these authorities to publish this table as the Sieve of Eratosthenes; especially as they are in some measure supported by passages of Nicomachus himself. But the Sieve of Eratosthenes was quite another thing.

* This manuscript seems to have contained the text of Nicomachus with Scholia in the margin. But the table evidently belongs to the Scholia, not to the text.

The Oxford editor hath annexed to his table, to explain the use of it, some detached passages, which he hath selected from the text of Nicomachus, and from a comment upon Nicomachus ascribed to Joannes Grammaticus. In these passages the difference between Prime and Composite numbers is explained, in many words indeed, but not with the greatest accuracy; and it is proposed to frame a kind of Table of all the odd numbers, from 3 to any given limit, in which the Composite numbers should be distinguished by certain marks*. The Primes would consequently be characterised, as far as the table should be carried, by being unmarked. But, upon what principles, or by what rule, such a table is to be constructed, is not at all explained. It is obvious that, in order to *mark* the Composite numbers, it is necessary to know which are such. And, without some rule to distinguish which numbers are Prime, and which are Composite, independent of any table in which they shall be distinguished by marks, it is impossible to judge, whether the table be true, as far as it goes, or to extend it, if requisite, to a further limit. Now it was the Rule by which the Prime numbers and the Composite might be distinguished, not a Table constructed we know not how, that was the invention of Eratosthenes, to which from its use, as well as from the nature of the operation, which

* Nicomachus and Joannes Grammaticus propose that these marks should be such, as should not only distinguish the composite numbers, but likewise serve to express all the divisors of every such number. It will be shewn, in a proper place, that this was no part of the original contrivance of the Sieve.

proceeds (as will be shewn) by a gradual extermination of the composite numbers from the arithmetical series 3. 5. 7. 9. 11. &c. infinitely continued, its author gave the name of the Sieve. I have thought it necessary to premise these remarks, to remove a prejudice, which I apprehend many may have conceived, as this beautiful and valuable edition of Aratus is in every ones hands, that this ill-contrived table, the useless work of some monk in a barbarous age, was the whole of the invention of the great Eratosthenes, and in justice to myself, that I might not be suspected of attempting to reap another's harvest.

I now proceed, to give a true account of this excellent invention; which, for its usefulness, as well as for its simplicity, I cannot but consider as one of the most precious remnants of Ancient Arithmetic. I shall venture to represent it according to my own ideas, not obliging myself to conform, in every particular, to the account of Nicomachus, which I am persuaded is in many circumstances erroneous. In stating the principles upon which the Operation of the Sieve was founded, he hath added observations upon certain relations of the odd numbers to one another, which are certainly his own, because they are of no importance in themselves, and are quite foreign to the purpose. Every thing of this kind I omit: and having stated what I take to have been the genuine Theory of Eratosthenes's method, cleared from the adulterations of Nicomachus, I deduce from it an operation of great simplicity, which solves the Problem in question with wonderful ease, and which,

because it is the most simple that the theory seems to afford, I scruple not to adopt as the original Operation of the Sieve, though nothing like it is to be found in Nicomachus; though, on the contrary, Nichomachus, and all his Commentators, would suggest an operation very different from it, and far more laborious. For the satisfaction of the curious and the learned, I have annexed a copy of so much of Nicomachus's treatise, as relates to this subject, with such corrections of the text, as it stands in the edition of Wiche-lius, printed at Paris ann. 1538, as the sense hath suggested to me, or I have thought proper to adopt, upon the authority of a manuscript preserved among those of Archbishop Laud, in the Bodleian Library; which, in this part, I have carefully collated. By comparing this with the account which I subjoin, every one will be able to judge how far I have done justice to the invention I have undertaken to explain.

P R O B L E M.

To find all the Prime Numbers.

The number 2 is a Prime number; but, except 2, no even number is Prime, because every even number, except 2, is divisible by 2, and is therefore Composite. Hence it follows, that all the Prime numbers, except the number 2, are included in the series of the odd numbers, in their natural order, infinitely extended; that is, in the series

3. 5. 7. 9. 11. 13. 15. 17. 19. 21. 23. 25. 27.
29. 31. 33. 35. 37. 39. 41. 43. 45. 47. 49. 51. &c.
Every

Every number which is not Prime, is a multiple of some Prime number, as Euclid hath demonstrated (Element. 7. prop. 33.) Therefore the foregoing series consists of the Prime numbers, and of multiples of the Primes. And the multiples, of every number in the series, follow at regular distances; by attending to which circumstance, all the multiples, that is, all the Composite numbers, may be easily distinguished and exterminated.

I say, the multiples of all numbers, in the foregoing series, follow at regular distances.

For between 3 and its first multiple in the series (9) two numbers intervene, which are not multiples of 3. Between 9 and the next multiple of 3 (15) two numbers likewise intervene, which are not multiples of 3. Again between 15 and the next multiple of 3 (21) two numbers intervene, which are not multiples of 3; and so on. Again, between 5 and its first multiple (15) four numbers intervene, which are not multiples of 5. And between 15 and the next multiple of 5 (25) four numbers intervene which are not multiples of 5; and so on. In like manner, between every pair of the multiples of 7, as they stand in their natural order in the series, 6 numbers intervene which, are not multiples of 7. Universally, between every two multiples of any number n , as they stand in their natural order in the series, $n-1$ numbers intervene, which are not multiples of n .

Hence may be derived an Operation for exterminating the Composite numbers, which I take to have been the Operation of the Sieve, and is as follows.

The Operation of the Sieve.

Count all the terms of the series following the number 3, by threes, and expunge every third number. Thus all the multiples of 3 are expunged. The first uncanceled number that appears in the series, after 3, is 5. Expunge the square of 5. Count all the terms of the series, which follow the square of 5, by fives, and expunge every fifth number, if not expunged before. Thus all the multiples of five are expunged, which were not at first expunged, among the multiples of 3. The next uncanceled number to 5 is 7. Expunge the square of 7. Count all the terms of the series following the square of 7, by sevens, and expunge every seventh number, if not expunged before. Thus all the multiples of 7 are expunged, which were not before expunged among the multiples of 3 or 5. The next uncanceled number which is now to be found in the series, after 7, is 11. Expunge the square of 11. Count all the terms of the series, which follow the square

3. 5. 7. 9. 11. 13. 15. 17. 19. 21. 23. 25. 27. 29. 31.
 33. 35. 37. 39. 41. 43. 45. 47. 49. 51. 53. 55. 57. 59.
 61. 63. 65. 67. 69. 71. 73. 75. 77. 79. 81. 83. 85. 87.
 89. 91. 93. 95. 97. 99. 101. 103. 105. 107. 109. 111. 113.
 115. 117. 119. 121. 123. 125. 127. 129. 131. 133. 135.
 137. 139. 141. 143. 145. 147. 149. 151. 153. 155. 157.

of 11, by elevens, and expunge every eleventh number, if not expunged before. Thus all the multiples of 11 are expunged, which were not before expunged among the multiples of 3, 5, and 7. Continue these expunctions, till the first uncanceled number that appears, next to that whose multiples have been last expunged, is such, that its square is greater than the last and greatest number to which the series is extended. The numbers which then remain uncanceled are all the Prime numbers, except the number 2, which occur in the natural progression of number from 1 to the limit of the series. By the limit of the series I mean the last and greatest number to which it is thought proper to extend it.

Thus the prime numbers are found to any given limit.

Nicomachus proposes to make such marks over the Composite numbers, as should shew all the divisors of each. From this circumstance, and from the repeated intimations both of Nicomachus, and his commentator Joannes Grammaticus *, one would be led to imagine, that the Sieve of Eratosthenes was something more than its name imports, a method of sifting out the Prime numbers from the indiscriminate mass of all numbers. Prime and Composite, and that, in some way or other, it exhibited all the divisors of every Composite number, and likewise shewed whether two or

* The Comment of Joannes Grammaticus is extant in manuscript in the Savilian Library at Oxford, to which I have frequent access, by the favour of the Reverend and Learned Mr. Hornsby, the Savilian Professor of Astronomy.

more Composite numbers were Prime or Composite with respect to each other. I have many reasons to think, that this was not the case. I shall as briefly as possible point out some of the chief, for the matter is not so important, as to justify my troubling the Society with a minute detail of them. First then, in the natural series of odd numbers, 3. 5. 7. &c. every number is a divisor of some succeeding number. Therefore if we are to have marks for all the different divisors of every Composite number, we must have a different mark for every odd number. Therefore we must have as many marks, or systems of marks, as numbers; and I do not see, that it would be possible, to find any more compendious marks, than the common numeral characters. This being the case, it would be impracticable to carry such a table as Nicomachus proposes, and his commentators have sketched, to a sufficient length to be of use, on account of the multiplicity of the divisors of many numbers, and the confusion which this circumstance would create*. It is hardly to be supposed, that Eratosthenes could overlook this obvious difficulty, though Nicomachus hath not attended to it. Eratosthenes therefore could not intend the construction of such a table.

In the next place, such a table not being had, Eratosthenes could not but perceive, that, the determining whether two or more numbers be Prime or Composite with respect to one another, is in all cases to be done more easily, by the direct method given by Euclid, than by

* The number 3465 hath no less than 22 different divisors.

the method of the Sieve. And he could not mean, to apply this method to a problem, to which another was better adapted.

Lastly, Eratosthenes could not mean, that the method of the Sieve should be applied to the finding of all the possible divisors of any Composite number proposed, because he could not be unacquainted with a more ready way of doing this, founded upon two obvious Theorems, which could not be unknown to him.

The Theorems I mean are these.

1st. If two Prime numbers multiply each other, the number produced hath no divisors but the two prime factors.

2d. If a Prime number multiply a Composite number, and likewise multiply all the divisors of that composite severally, the numbers produced by the multiplications of these divisors will be divisors of the number produced by the first multiplication: And the number produced by the first multiplication will have no divisors, but the two factors, the divisors of the Composite factor, and the numbers made by the multiplication of these divisors by the Prime factor severally.

The method of finding all the divisors of any Composite number, delivered by Sir Isaac Newton in the *Arithmetica Universalis*, and by Mr. Maclaurin in his *Treatise of Algebra*, may be deduced from these propositions, as every mathematician will easily perceive. This method requires indeed that the least prime divisor should be previously found; and, if the least prime divisor should happen to be a large number, as it is not assignable by any general method, the

investigation of it by repeated tentations may be very tedious. A table therefore of the odd numbers *, in which the Composite numbers should each have its least Prime divisor written over it, would be very useful. But Nichomachus's project of framing a table in which each Composite number should have *all* its divisors written over it, is ridiculous and absurd, on account of the insuperable difficulties which would attend the execution of it.

Feb. 7, 1772.

S. Horsley.

* A table of the odd numbers would be sufficient: for the number 2 is the least prime divisor of every even number; and it is easy, even in the largest numbers, to try whether they are divisible by 2. In our method of notation, this may always be known, by observing the last figure in the expression of the number proposed.

EXCERPTA QUÆDAM

E X

Arithmeticâ Nicomachi

Ad Cribrum Eratosthenis pertinentia.

Ἡ ὃ τέτων ἡμέσεις (α), ὑπὸ Ερατοσθένους, καλεῖται Κόσκινον· ἐπειδὴ ἀναπεφυρμένους τὰς περισσεύς λαβούνας καὶ ἀδιακρίτους, ἐξ αὐτῶν [τὰ διαφέροντα ἀλλήλων ἔδη] (β) ταύτη τῇ τῆς ἡμέσεως (γ) μεθόδῳ διαχωρίζομεν, ὡς δι' ὄργανον ἢ κοσκίνον τινός· καὶ ἰδίᾳ μὲν τὰς πρώτας καὶ ἀσυνθέτους, χωρὶς ὃ τὰς μίκτους εὐρίσκουμεν. Ἐστὶ ὃ ὁ τρόπος τῶ Κοσκίνου τοιοῦτος. Ἐκθέμενος τὰς ἀπὸ τριάδος πάντας ἐφεξῆς περισσεύς, ὡς δυνατὸν μάλιστα ἐπὶ μήκιστον εἶχον, ἀρχάμενος ἀπὸ τῶ πρώτου, ἐπισκοπῶ τίνες οἷός τε εἶναι μέμεινεν ἕκαστος καὶ εὐρίσκω δυνατὸν ὄντα τὸν πρώτον, ἦτοι τὸν γ', τὰς δύο μέσας διαλείποντας (δ) μέμεινεν, μέχρις ἢ προχωρῶν ἐθέλωμεν (ε). ἔχ' ὡς ἔτυχεν ὃ, καὶ εἰκῇ, μέμεινεν, ἀλλὰ τὸν μὲν πρώτως αὐτῶν κείμενον, τὸν ἑστὶ τὸν ἀφ' αὐτοῦ τὰς δύο μέσας διαλεί-

(α) Malletem εὔρεσις, etsi, ne quid diffimulem, lectioni receptæ adstipulatur Boethii interpretatio.

(β) Voces uncis inclusas conjecturâ supplevi; quin et sequentium ordinem paululum immutavi, pro τῇ ἡμέσεως μεθόδῳ ταύτῃ, scribendo ταύτη τῇ κ. τ. λ.

(γ) Vocem ἡμέσεως hic loci retinendam censeo. Locum integrum sic interpretor. “Suam horum indaginem Eratosthenes, Cribrum vocavit. Propterea quod imparibus universis, nullo generum discrimine, in medio collocatis, ipsam procreationem continuam, quo tradidit ille modo, insequendo [id est, procreationis continuæ, Eratosthenis modo, exploratâ lege] species diversas seorsim sistimus, cribro tanquam separatas.”

(δ) Cod. MS. habet διαλείποντας. Wechelius παραλείποντας.

(ε) Ex Cod. MS. pro ἐθέλωμεν.

πονη(*f*), καὶ τὴν τῇ πρῶτῃ ἐν τῷ εἰχῶ καμένην ποσότητα
 μετρήσει· τῷτ' ἐστὶ καὶ τὴν ἐαυτῆς, τρεῖς γὰρ· τὸν δ' ἀπ'
 ἐκείνης δύο διαλείποντα, καὶ τὴν τῇ δευτέρῃ τεταγμένην,
 πεντάκις γὰρ· τὸν ὃ περαιτέρω πάλιν δύο διαλείποντα, καὶ
 τὴν τῇ τρίτῃ τεταγμένην, ἐπτάκις γὰρ· τὸν ὃ ἐτι περαιτέρω
 ὑπὲρ δύο κείμενον, καὶ τὴν τῇ τέταρτῃ τεταγμένην, ἐννεάκις
 γὰρ· καὶ ἐπ' ἀπειρον τῷ αὐτῷ τρόπῳ. Εἴτα μετὰ τῶτον, ἀπ'
 ἄλλης ἀρχῆς, ἐπὶ τὸν δεύτερον ἐλθὼν, σκοπῶ τίνος οἷός τε
 ἐστὶ μετρεῖν· καὶ εὐρίσκω πάντας τὰς τέσσαρας (*g*) διαλείποντας·
 ἀλλὰ τὸν μὲν πρῶτον, καὶ τὴν ἐν τῷ εἰχῶ πρῶτῃ
 τεταγμένην ποσότητά· τρεῖς γὰρ· τὸν ὃ δεύτερον, καὶ τὴν τῇ
 δευτέρῃ· πεντάκις γὰρ· τὸν ὃ τρίτον, καὶ τὴν τῇ τρίτῃ·
 ἐπτάκις γὰρ· καὶ τῷτο ἐφεξῆς αἰεὶ. Πάλιν ὃ ἀνωθεν, ὁ
 τρίτος, ὁ ζ', τὸ μετρεῖν* παραλαβὼν, μετρήσει τὰς ἐξ δια-
 λείποντας· ἀλλὰ τὸν μὲν πρώτιστον, καὶ τὴν τῇ γ' (*b*)
 ποσότητά, πρῶτῃς κειμένη· τὸν ὃ δεύτερον καὶ τὴν τῇ ε'
 δευτεροταγῆς γὰρ ἔστω (*i*). τὸν ὃ τρίτον, καὶ τὴν τῇ ζ',
 τρίτην γὰρ ἔχει (*k*) ἔστω τάξιν ἐν τῷ εἰχῶ. καὶ, καὶ τὴν
 αὐτὴν ἀναλογίαν, δι' ὅλον (*l*) ἀπαραποδίστως (*m*) προχωρήσει
 σοι τῷτο, ὥστε τὸ μὲν μετρεῖν διαδέξον, καὶ τὴν ἐν τῷ
 εἰχῶ αὐτῶν ἐγκαμμένην τάξιν· τὸ ὃ πόσους διαλείποντας,

(*f*) Locum in Editione Wechelii corruptum, in Cod. MS. mutilum & turbatum, conjecturâ, prout potui, sanatum dedi. Editio Wechelii habet τὸν τὰς δύο μίσας ὑπερβαίνοντα. Codex MS. τὸν δύο. τῇ ἐστὶ τὸν τρία.

(*g*) Conjecturâ, pro τετράδι.

(*b*) Litera numeralem γ', conjecturâ posui pro voce τρία.

(*i*) Restitui ex Cod. MS pro δ'ν', quæ est Wechelii lectio.

(*k*) Particulam καὶ omisi.

(*l*) Wechelium sequor. Cod. MS. habet λεγς, sensu, ut videtur, nullo.

(*m*) Ex Cod. MS. pro ἀπαραμπόδισον.

* Conjecturâ pro μετρον.

καὶ τὴν ἀπὸ δυνάδεω ἐπ' ἄπειρον εὐτακτον τῶν (n) ἀρίων
προκοπὴν, ἢ καὶ τὴν τ' χωρὶς διπλασίασιν καθ' ἣν ὁ
μετῶν τέτακ'· τὸ ὅ ποσάκις, καὶ τὴν τῶν ἀπὸ τετράδεω
περιστῶν εὐτακτον ἐπ' ἄπειρον (o) προχώρησιν (p). Ἐὰν ἔν
σημείοις τισὶν ἐπισιζῆς τὰς ἀριθμῶν, εὐρήσεις τὰς μετὰ-
λαμβάνουσας τὸ μετρεῖν, ἔτε ἅμα πάντας τ' αὐτὸν ποιε
μετρεῖν, ἔστι ὅτε ἔδὲ δύο τ' αὐτὸν· ἔτε πάντας ἀπλῶς
τὰς ἐκκειμῶνς ὑποπίπνουσας μέτρῳ τινὶ αὐτῶν. ἀλλὰ
τινας μὲν παντελῶς διαφεύγουσας τὸ μετρηθῆναι ὑφ' ἑτισοσῶν·
τινας ὅ ὑφ' ἐνὸς μόνου μετρεμῶνς· τινὰς ὅ ὑπὸ δύο, ἢ καὶ
πλειόνων. Οἱ μὲν ἔν μηδαμῶς (q) μετρηθέντες, ἀλλὰ δια-
φυγόντες τῷτο, πρῶτοι εἰσὶ καὶ ἀσύνθετοι, ὡς ὑπὸ κοσκήνης
διακρηθέντες. οἱ ὅ ὑφ' ἐνὸς μόνου μετρηθέντες, καὶ τὴν
ἐαυτῶ (r) ποσότητα, ἐν μόνον μόριον ἑτερώνυμον ἔχουσι πρὸς
τῷ παρωνύμῳ. οἱ δὲ ὑφ' ἐνὸς μὲν (s), ἑτέρου δὲ ποσότητι, καὶ
μὴ τῇ ἐαυτῶ, ἢ ὑπὸ δύο ὁμῶς μετρηθέντες, πλείονα ἔχουσι τὰ
ἑτερώνυμα μέρη πρὸς τῷ παρωνύμῳ. τῷτοι ἔν ἔσονται

(n) Conjectura pro τὴν.

(o) Voces ἐπ' ἄπειρον ex Cod. MS. restitui.

(p) Nempe series numerorum imparium 3, 5, 7, 9, &c. infinite protensa, cum numeros impares universos contineat, imparis cujusvis multiplices omnes impares necessario complectitur. Est igitur n numerus quilibet impar. In serie 3, 5, 7, &c. infinite protensâ, habes numeros omnes $n \times 3$, $n \times 5$, $n \times 7$, $n \times 9$, &c. Et cum seriei ea Lex sit & Condicio, ut naturali ordine numeri impares sequantur, & minor omnis numerus majorem præcedat, fieri nequit, quin multiplices numeri n eum inter se ordinem fervent, ut minor quisque majorem præcedat. Primus igitur erit $n \times 3$, secundus $n \times 5$, tertius $n \times 7$, & universim, $n \times m$ eum habiturus est, inter multiplices, locum, quem numerus m in serie.

(q) Ex Cod. MS. vice ἑδαμῶς, quæ Wechelii lectio est.

(r) Conjecturâ pro ἐαυτῶν.

(s) Particulam μὲν ex Cod. MS. restitui.

δεύτεροι καὶ σύνθετοι. Τὸ δὲ τρίτον μέρ^{ος}, το κοινὸν ἀμφοτέρων, ὁ καθ' ἑαυτὸ μὴ δὲ δεύτερον καὶ σύνθετον, πρὸς ἄλλο δὲ πρῶτον καὶ ἀσύνθετον, ἔσονται ἀποτελέματοι ἀριθμοὶ, καὶ αὐτὰ τὴν ἑαυτῶν ποσότητα πρώτης καὶ ἀσυνθέτης μετρήσαντες τινός, εἴτις [τέτῳ τῷ τρόπῳ] (t) ἀριθμ^{οῦ}, συγκρίνοισι πρὸς ἄλλον ὡσαύτως τὴν ἡμέσιν ἔχουσα. ὥστε ὁ 5, ἐφ' ᾧ γὰρ ἐκ τῆς γ' (u) κατὰ τὴν ἑαυτῶν ποσότητα μετρήσαντες τρίς γὰρ εἰ συγκρίνοισι πρὸς τὴν κ' εἰ ἐφ' ᾧ γὰρ καὶ 5τ^{ος} (x) ἐκ τῆς ε', καὶ αὐτὰ τὴν ἑαυτῶν ποσότητα μετρήσαντες πεντάκις γὰρ κοινὸν μέτρον τέτοις εἶναι, εἰ μὴ μόνῃ ἢ Μονάδι.

(t) Voces τέτῳ τῷ τρόπῳ conjecturâ supplevi.

(u) Literam numeralem γ' pro voce τρίτη quæ apud Wechelium legitur, ex Cod. MS. restitui.

(x) Voces γὰρ καὶ 5τ^{ος} ex Cod. MS. restitui.

Ex Arithmetica Boethii.

Lib. I. c. xvii.

GENERATIO autem ipsorum atque ortus hujusmodi investigatione colligitur, quam scilicet Eratosthenes Cribrum nominabat; quod cunctis imparibus in medio collocatis, per eam, quam tradituri sumus, artem, qui primi, quive secundi, quique tertii generis videantur esse distinguuntur. Disponantur enim a ternario numero cuncti in ordinem impares, in quamlibet longissimam porrectionem 3. 5. 7. 9. 11. 13. 15. 17. 19. 21. 23. 25. 27. 29. 31. 33. 35. 37. 39. 41. 43. 45. 47. 49. His igitur ita dispositis, considerandum, primus numerus quem eorum, qui sunt in ordine positi, primum metiri possit: sed, duobus præteritis, illum, qui post eos est positus, mox metitur: et, si post eundem ipsum quem mensus est, alii duo transmissi sunt, illum, qui post duos est, rursus metitur: et, eodem modo si duos quis reliquerit, post eos qui est, a primo numero metiendus est; eodemque modo, relictis semper duobus, a primo, in infinitum pergentes metientur. Sed id non vulgo neque confuse. Nam primus numerus illum, qui est post duos secundum se locatos, per suam quantitatem metitur: ternarius enim numerus ter ^a 9 metitur. Si autem post novenarium duos reliquero, qui mihi post illos incur-

^a Conjecturâ pro *tertio*.

rit, a primo metiendus est, per secundi imparis quantitatem; id est, per quinarium: nam si post 9 duos relinquam, id est 11 & 13, ternarius numerus 15 metietur, per secundi numeri quantitatem, id est, per quinarium; quoniam numerus ternarius 15 quinquies metitur. Rursus, si a quindenario inchoans duos intermisero, qui posterior positus est, ejus primus numerus mensura est, per tertii imparis pluralitatem: nam si post 15 intermisero 17 & 19, incurrit 21, quem ternarius numerus secundum septenarium metitur; 21 enim numeri ternarius septima pars est: atque hoc in infinitum faciens, reperio primum numerum, si binos intermisero, omnes sequentes post se metiri, secundum quantitatem positorum ordine imparium numerorum. Si vero quinarium numerus, qui in secundo loco est constitutus, velit^b quis, cujus prima ac deinceps sit mensura, invenire, transmissis quatuor imparibus, quintus ei quem metiri possit, occurrit. Intermittantur enim quatuor impares, id est, 7 & 9, & 11 & 13, post hos est quintus decimus quem quinarium metitur, secundum primi scilicet quantitatem, id est, ternarii; quinque enim 15 ternarii metiuntur: ac deinceps, si quatuor intermittat, eum qui post illos locatus est, secundus, id est, quinarium, sui quantitate metitur: nam post quindecim intermissis 17 & 19, & 21 & 23, post eos 25 reperio, quos quinarium scilicet numerus suâ pluralitate metitur; quinquies enim quinario multiplicato, 25 succrescunt; si vero post hunc quilibet quatuor intermittat, eâdem ordinis servatâ

^b Conjecturâ pro *vel*.

^c Conjecturâ pro *tertio*.

constantia, qui eos sequitur, secundum tertii, id est, septenarii numeri summam, a quinario metitur: atque hæc est infinita processio. Si vero tertius numerus quem metiri possit exquiratur, sex in medio relinquentur; & quem septimum ordo monstraverit, hic per primi numeri, id est, ternarii quantitatem metiendus est: et post illum, sex aliis interpositis, quem post eos numeri series dabit, per quinarium, id est, per secundum, tertium mensura percurrent: si vero alios rursus sex in medio quis relinquat, ille, qui sequitur, per septenarium ab eodem septenario metiendus est; id est, per tertii quantitatem; atque hic usque in extremum ratus ordo progreditur. Suscipient ergo metiendi vicissitudinem, quemadmodum sunt in ordine naturaliter impares constituti: metientur autem, si per pares numeros, a binario inchoantes, positos inter se impares, ratâ intermissione, transilient; ut primus duos, secundus quatuor, tertius sex, quartus octo, quintus decem^d: vel si locos suos conduplicent, & secundum duplicationem terminos intermittant; ut ternarius, qui primus est numerus, & Unus, omnis enim primus Unus est, bis locum suum multiplicet, faciatque bis unum; qui cum duo sint, primus duos medios transeat. Rursus secundus, id est, quinarium, si locum suum multiplicet, 4 explicabitur: hic quoque quatuor^e intermittat. Item si septenarius, qui tertius est, locum suum duplicet, sex creabit; bis enim 3 senarium jungunt: hic ergo in ordine^f sex relinquat. Quartus quoque, si locum

^d Conjecturâ restitui pro 12.

^e Conjecturâ pro 4.

^f Conjecturâ pro *ordinem*.

suum duplicet, 8 succrescent; ille quoque octo tranfiliat: atque hoc quidem in cæteris perspiciendum. Modum autem mensuræ, secundum ordinem collocatorum, ipsa series dabit. Nam primus primum quem numerat, secundum primum numerat^g, id est, secundum se; & secundum primum quem numerat, per secundum numerat^g, & tertium per tertium, & quartum item per quartum. Cum autem secundus mensuram^h susceperit, primum quem numerat secundum primum metitur; secundum vero quem numerat per se, id est, per secundum; & tertium per tertium: & in cæteris eadem similitudine mensuræ constabit. Illosⁱ ergo si respicias, vel qui alios mensi sunt, vel qui ipsi ab aliis metiuntur, invenies omnium simul communem mensuram esse non posse, neque ut omnes quemquam alium simul numerent; quosdam autem ex his ab alio posse metiri, ita ut ab uno tantum numerentur^k; alios vero, ut etiam a pluribus; quosdam autem, ut præter Unitatem eorum nulla mensura sit. Qui ergo nullam mensuram præter Unitatem recipiunt, hos Primos & Incom-

^f Conjecturâ pro 8.

^g Pro *numerat* mallet in utroque loco, *metitur*, ut aliud sit *numerare*, aliud *metiri*, & sensus sit, "That which the first number [of the Series] counts the first [of its multiples], it measures by the first [of the Series], i. e. by itself. That which it counts the second [of its multiples], it measures by the second [number in the Series]." Sic enim infra legimus de Numero ordine secundo, "primum quem numerat secundum primum metitur."

^h Conjecturâ, pro *mensuram*.

ⁱ Conjectura, pro *alios*.

^k Ang. "But so as to be counted in among the multiples of one number only."

positos judicamus; qui vero aliquam mensuram præter Unitatem, vel alienigenæ partis vocabulum fortiuntur, eos pronunciemus Secundos atque Compositos. Tertium vero illud genus, per se Secundi & Compositi, Primi vero & Imcompositi ad alterutrum comparati, hæc inquisitor ratione reperiet. Si enim quoslibet primos ¹ numeros, secundum suam in semetipfos multiplices quantitatem, qui procreantur, ad alterutrum comparati, nullâ mensurâ communione junguntur: 3^m enim & 5, si multiplices, 3 ter ⁿ 9 faciunt, & quinquies 5 reddunt 25. His igitur nulla est cognatio communis mensuræ. Rursus 5 & 7 quos procreant, si compares, hi quoque incommensurabiles erunt: quinquies enim 5, ut dictum est, 25, septies 7 faciunt 49; quorum mensura nulla communis est, nisi forte omnium horum procreatrix & mater Unitas °.

¹ Conjectura pro *illos*.

^m Conjecturâ, pro *tres*. ⁿ Conjecturâ pro *tres tertio*.

° Sed cave credas, Lector, numeros inter se primos nullos dari præter Primorum Quadratos.